Elastic creep of stressed solids due to time–dependent changes in elastic properties

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The suggestion is made that creep in solids can occur by time-dependent changes in elastic properties. Specific mechanisms include cavity formation and growth, crack nucleation and growth, grain boundary migration in polycrystalline solids with elastically anisotropic grains and the redistribution of the individual phases within a composite. Creep rates by these four mechanisms are analysed and discussed for simple mechanical models. Recommendations are made for the interpretation of creep data in order to clearly separate the contribution of elastic creep from the total creep deformation.

1. Introduction

Mechanically stressed solids can exhibit time-dependent deformation, referred to as creep. This can occur by a number of well accepted mechanisms such as dislocation glide [1] and climb [2], Nabarro–Herring [3, 4] and Coble creep [5] and diffusion accommodated grain-boundary sliding [6].

The purpose of this communication is to suggest the existence of an additional mechanism of creep which may make a minor, or even major, relative contribution to the total observed creep deformation. This mechanism is based on microstructural changes during creep deformation which cause time-dependent decreases in the elastic properties. Under stress, such changes in elastic behaviour will lead to time-dependent changes in the elastic strain, superposed on any other creep strain due to other creep processes. These writers have chosen to refer to such creep due to changes in elastic properties as "elastic" or "compliance" creep.

Elastic creep can occur by a number of different mechanisms. The formation of cavities in many materials undergoing creep is a well-known phenomenon [7–10]. It is also well known that cavities will reduce the elastic moduli of solids [11, 12]. For this reason, cavity formation and growth should lead to elastic creep.

The existence of crack growth under stress is well recognized. Cracks have a profound effect on elastic properties [13–15]. Therefore, crack formation and growth should also lead to elastic creep. Although not defined as elastic creep, the non-linear deformation of a heavily microcracked ceramic [16, 17] at room temperature was attributed [18] to crack growth by stress corrosion. Also, elastic creep is implicit in the formulations of Evans and Rana [19] and Evans [20] for the creep due to the nucleation and growth, respectively, of crack-like cavities. The order of magnitude difference in creep rates in tension and compression observed for reaction-sintered and hot-pressed silicon nitride was thought to be because, in tension, grain-boundary crack formation was the rate-controlling process [21, 22]. Possibly, in these latter studies, part of the observed creep represents elastic creep by the nucleation and growth of such cracks.

The elastic properties of polycrystalline materials with elastically anisotropic grains depend on the degree of preferred orientation of the grains [23–25]. Migration of grain boundaries is a well-known phenomenon. For these reasons, grain-boundary migration under load, in such a manner that a greater proportion of the grains are aligned with their direction of lower Young's modulus as much as possible parallel to the applied stress,

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represents a further possible mechanism of elastic creep.

Finally, the elastic properties of composites depend strongly on the nature of the distribution of the individual components within the composite [26]. For this reason any mechanism which alters this phase distribution under stress should lead to elastic creep as well.

It is the purpose of this communication to present brief analyses for elastic creep by the four mechanisms above, based on appropriate mechanical models for which literature equations for the elastic properties are available.

2. Analysis

2.1. Elastic creep by cavity formation and growth

A solid will be considered with a fractional volume P of N spherical cavities per unit volume of solid. All cavities have equal volume V, with $NV = P$. The cavity size is assumed to be very small compared to the size of the unit cube.

Solutions for the shear and bulk moduli of elasticity of solids with spherical cavities were obtained by Hashin [12] from which an expression for Young's modulus is readily derived. The complete expression is relatively cumbersome. For simplicity, for values of Poisson's ratio, ν of the order of 0.2 and 0.3, and a dilute concentration of cavities, to a very good approximation, Young's modulus (E) can be written [11, 12]

$$E \simeq E_0(1-2P), \qquad (1)$$

where E_0 is Young's modulus of the solid without cavities.

For a uniaxial stress (σ), the elastic strain (ϵ) is

$$\epsilon \simeq \sigma/E_0(1-2P), \qquad (2a)$$

which for $2P \ll 1$, yields

$$\epsilon \simeq \sigma(1+2P)/E_0. \qquad (2b)$$

Differentiating with respect to time results in the rate of change of strain (i.e. rate of elastic creep)

$$\dot{\epsilon} \simeq 2\sigma\dot{P}/E_0 \qquad (3)$$

writing $\sigma/E_0 = \epsilon_0$, the elastic strain without cavities, yields

$$\dot{\epsilon}/\epsilon_0 \simeq 2\dot{P} \qquad (4)$$

which indicates that the relative rate of change of the elastic strain is twice the rate of change of the fractional volume of cavities.

Since $P = NV$, Equation 3 can be written

$$\dot{\epsilon} \simeq 2(\sigma/E_0)(N\dot{V} + V\dot{N}) \qquad (5)$$

which simultaneously reflects the effect of the growth of existing cavities and the nucleation of cavities on the elastic creep rate. For a distribution of cavity sizes and a size dependent growth rate, Equation 5 can be written in integral form.

The stress exponent of the elastic creep rate due to cavity formation and growth can be estimated from the study of Miller and Langdon [27], who analysed literature data for the density changes during creep for a number of metals. From these results it can be shown that the rate of change of fractional volume of growing and nucleating cavities ($\dot{P}$) can be written (in the present notation)

$$\dot{P} = B(\epsilon_t/d)(\sigma/G)^q \exp(-Q_{gb}/RT), \qquad (6)$$

where ϵ_t is the total creep strain, d is the grain size, G is the shear modulus, Q_{gb} is the activation energy for grain-boundary diffusion, R is the gas constant, T is the absolute temperature and B and q are constants with $q \simeq 2$ to 3.

Substitution of Equation 6 into Equation 4 results in

$$\dot{\epsilon} = K\sigma^r, \qquad (7)$$

where for a given temperature, K is a constant obtainable from Equations 4, 5 and 6 and $r = q+1$, or $3 < r < 4$. This indicates the non-linear nature of elastic creep by cavity formation and growth.

2.2. Elastic creep by crack growth

A uniaxially stressed solid contains N penny-shaped cracks per unit volume. All cracks have equal radius of revolution, r and are oriented perpendicular to the applied stress. For such a solid with cracks, the effective Young's modulus is [28]

$$E_{eff} = E_0(1 + 16(1-\nu^2)Nr^3/3)^{-1}, \qquad (8)$$

where ν is Poisson's ratio.

For a uniaxial stress (σ) the elastic strain (ϵ) is

$$\epsilon = \sigma[1 + 16(1-\nu^2)Nr^3/3]/E_0. \qquad (9)$$

Differentiation with respect to time yields the rate of elastic creep

$$\dot{\epsilon} = 16(1-\nu^2)\sigma r^2(3N\dot{r} + r\dot{N})/3E_0, \qquad (10)$$

where $\dot{r}$ is the rate of crack growth and $\dot{N}$ is the rate of crack nucleation. For a distribution in crack sizes Equation 9 can be written in integral form.

In order to establish the stress exponent for elastic creep by crack growth, extensive literature data have shown that the rate of crack growth at a given temperature can be described by

$$\dot{r} = AK_{\mathrm{I}}^{n}, \quad (11)$$

where A and n are constants for a given material. The values of n range from near 10 for hot-pressed silicon nitride [29], and near 18 for soda–lime–silica glass [30] at room temperature in most environments to as high as 200 for graphite [31] in room air. In Equation 11, K_{I} is the mode I stress intensity factor given by

$$K_{\mathrm{I}} = Y\sigma r^{1/2}, \quad (12)$$

where Y is a constant for a given crack and specimen geometry.

Substitution of Equation 12 into Equation 11 yields for constant crack density, $N(\dot{N} = 0)$

$$\dot{\epsilon} = [16(1-\nu^2)NY^nA/E_0]\,\sigma^{n+1}r^{n/2+2} \quad (13)$$

which indicates that elastic creep by crack growth is highly non-linear. Since the material will fracture at $K = K_{\mathrm{Ic}}$, the critical stress intensity factor, elastic creep by crack growth is expected to occur primarily at stress levels immediately below the fracture stress. This latter conclusion agrees with the findings of Hasselman *et al.* [32], who compiled modified creep deformation maps [33] for polycrystalline aluminas, which included creep by crack growth as a creep mechanism.

2.3. Elastic creep by grain-boundary migration

A polycrystalline solid is considered with an idealized microstructure consisting of thin parallel single-crystal slabs. Alternate slabs have values of Young's modulus E_1 and E_2 corresponding to the maximum and minimum values of Young's modulus of the single crystal of the material, respectively. The model contains N grain boundaries per unit volume.

2.3.1. Grain boundary migration perpendicular to the stress

In this case, the plane of the grain boundaries is oriented parallel to the stress. The effective Young's modulus for this configuration corresponds to Paul's [34] upper bound (E_+) on the elastic moduli of a two-component composite. Assuming equality of Poisson's ratio for the different slabs, Young's modulus is

$$E_+ = V_1E_1 + (1-V_1)E_2, \quad (14)$$

where V_1 is the volume fraction of slabs with Young's modulus, E_1.

For a uniaxial stress, the elastic strain is

$$\epsilon = \sigma[V_1E_1 + (1-V_1)E_2]^{-1}. \quad (15)$$

Differentiating with respect to time yields the elastic creep rate

$$\dot{\epsilon} = -\sigma(E_1 - E_2)\dot{V}_1[V_1E_1 + (1-V_1)E_2]^{-2}. \quad (16)$$

The direction of grain-boundary migration will be such as to decrease the fraction of material with the higher Young's modulus, E_1, i.e. equivalent to decreasing, V_1. For the migration of N grain boundaries at a rate, $\dot{d}$

$$\dot{V}_1 = -N\dot{d} \quad (17)$$

which upon substitution in Equation 16 with the aid of Equation 14, yields

$$\dot{\epsilon} = \sigma N\dot{d}(E_1 - E_2)/E_+^2. \quad (18)$$

Equation 18 suggests that the elastic creep rate by boundary migration perpendicular to the direction of stress is directly proportional to the degree of elastic anisotropy ($E_1 - E_2$), the number and rate of migrating boundaries.

The present writers are not aware of literature solutions for stress-activated grain-boundary migration. Nevertheless, since such migration is expected to occur by a diffusional process, the rate of migration is expected to exhibit a stress exponent at least equal to unity or higher. For this reason, the elastic creep rate given by Equation 18 is expected to be proportional to the quadratic or higher power of stress.

2.3.2. Grain-boundary migration parallel to the stress

The mechanical model considered for this case is identical to the one considered in Section 2.3.1, with the exception that the stress direction is perpendicular to the plane of the slabs. Assuming identical Poisson's ratio for all slabs, Young's modulus for this configuration and stress direction corresponds to the lower bound [34], E_- on Young's modulus of a two-component composite

$$E_-^{-1} = V_1/E_1 + (1-V_1)/E_2. \quad (19)$$

For a uniaxial stress (σ) the elastic strain is

$$\epsilon = \sigma[V_1/E_1 + (1 - V_1)/E_2]. \quad (20)$$

Differentiation with respect to time yields for the creep rate

$$\dot{\epsilon} = -\sigma\dot{V}_1(1/E_2 - 1/E_1). \quad (21)$$

The direction of grain-boundary migration is such as to cause a decrease in V_1. For N boundaries migrating at a rate, $\dot{d}$

$$\dot{V}_1 = -N\dot{d} \quad (22)$$

which upon substitution into Equation 21 yields

$$\dot{\epsilon} = \sigma N\dot{d}(E_1 - E_2)/E_1E_2. \quad (23)$$

In analogy to the discussion in Section 2.3.1 elastic creep by grain-boundary migration parallel to the stress is expected to be proportional to the stress raised to the power two or higher.

Comparison of Equations 18 and 23 shows that elastic creep by grain-boundary migration parallel and perpendicular to the applied stress is governed by the same variables, with the exception that the creep rate by boundary migration perpendicular to the stress is an inverse function of the instantaneous value of the Young's modulus of the structure (i.e. the value of V_1). In contrast, for elastic creep by boundary migration parallel to the stress, the creep rate is independent of V_1.

2.4. Elastic creep by the redistribution of components within a composite

For the analysis of this mechanism of elastic creep, the composite geometry consists of a flat plate containing a dilute volume fraction (V_2) of elliptical inclusions with identical orientation. The composite is subjected to a pure shear stress, τ, on a plane within the plate at an angle of 45° to the major axis.

For this direction of stress the shear modulus (G) can be written

$$G = G_0(1 + \alpha V_2). \quad (24)$$

For conditions of plane strain the constant α in Equation 24 is [35]

$$\alpha = -\frac{4(K-1)(1-\nu)}{(1-K)(R-1)^2/(R+1)^2-(K(3-4\nu)+1)}, \quad (25)$$

where $K = E_2/E_1$ the ratio of the Young's moduli of inclusion and matrix and R is the ratio of the major-to-minor axis of the elliptical inclusions.

For a shear stress τ, the shear strain

$$\gamma = \tau/G_0(1 + \alpha V_2). \quad (26)$$

For constant V_2, the elastic creep rate

$$\dot{\gamma} = -(\tau/G_0)V_2(1 + \alpha V_2)^{-2}\dot{\alpha}, \quad (27)$$

writing

$$\dot{\alpha} = (\partial\alpha/\partial R)\dot{R} \quad (28)$$

a differentiation of Equation 25 with respect to R, yields

$$\dot{\alpha} = \frac{-16\dot{R}(1-K)^2(1-\nu)(R^2-1)/(R+1)^4}{[(1-K)(R-1)^2/(R+1)^2-(K(3-4\nu)+1)]^2} \quad (29)$$

which upon substitution into Equation 27 results in the required creep rate.

For $R > 1$ and $0 < K < \infty$, a positive creep rate results for $\dot{R} > 0$. This implies that elastic creep can occur by an increase in the aspect ratio i.e. an increase of the major axis and corresponding decrease in the minor axis. For a circular inclusion with $R = 1$, an unstable equilibrium results as inclusion deformation can occur by increasing both the major and minor axis.

3. Discussion

The relative contribution to the total creep deformation by each of the four mechanisms of elastic creep analysed above is expected to differ significantly. The compilation of literature data of Miller and Langdon [27] suggests that for many metals the total cavity volume fraction rarely exceeds 1%. At least for these cases the elastic creep rate is expected to be a small fraction of the total observed creep rate. However, if cavity growth and the corresponding elastic creep rate is a substantial fraction of the total non-linear deformation, care must be taken in the analysis of the data. Any elastic strain must be subtracted from the total observed strain. For instance, this will effect the value of the strain (ϵ) factor in the Equation 6 of Miller and Langdon [27] as well as the appropriate exponent.

Elastic creep by crack growth in brittle materials may well be the only mechanism of creep deformation at low or moderate temperature levels. At least for aluminum oxide, this was demonstrated by Hasselman *et al.* [32] who from crack growth data inferred the existence of creep by crack growth at temperature ranges over which diffusional creep mechanisms (i.e. Nabarro–Herring, Coble) were negligible. It was shown that this effect is more likely to occur in coarse-grained than in fine-grained alumina, resulting in a grain-size effect opposite to that for Nabarro–Herring and Coble creep. Possibly, the observations of Birch *et al.* [22] on the role of cracks in the creep of silicon nitrides may be interpreted in terms of elastic creep. Certainly, for temperature ranges over which stress corrosion effects occur (i.e. near room temperature) creep by crack growth is the only mechanism of creep. Since the stress range over which elastic creep by crack growth occurs is expected to be an inverse function of the function of the exponent n in Equation 11, creep by this mechanism is more likely to be observed for materials with low rather than high values of n.

The existence of elastic creep by crack growth is critical in the interpretation of experimental data for non-linear deformation not only in creep but also in single-cycle loading conditions. As an example, the excessively cracked microstructure of aluminum oxide published by Kingery *et al.* [36] may be considered. From the crack density and size it can roughly be estimated that during the deformation Young's modulus was reduced to at least one half to one third of the value of the crack-free aluminum oxide prior to deformation. For this reason, during deformation under a given stress, as the result of the formation of cracks, the aluminium oxide underwent an elastic strain some two to three times the elastic strain of the original non-cracked material. In general, it is critical to note then, that any observed non-linearity of a brittle material, at least in part could be the result of a change in elastic behaviour during deformation. In order to obtain a correct measure of the strain due to the non-linear (plastic or diffusional flow) an elastic strain must be subtracted from the total observed strain. The interpretation of creep data could become particularly complex if elastic creep by crack growth were coupled with crack-accelerated creep by another mechanism analysed by Weertman [37].

Regardless of the details considered, the total strain of elastic creep by crack growth is expected to be of the order of a small multiple (2 to 3) of the initial elastic strain to which the material is subjected during initial loading.

Elastic creep by grain-boundary migration is expected to be a transient phenomenon and contribute primarily to the initial stages of creep deformation. In the mechanical model considered for the analysis of this creep behaviour the transient nature arises because the unfavourably oriented grain will be consumed by the favourably oriented grains. Once this process is completed, no further grain-boundary migration and corresponding elastic creep deformation will occur. In actual polycrystalline materials, the grain boundaries exist in the form of a three dimensional network with the boundaries between two adjacent grains pinned at triple points. This pinning permits elastic creep by grain-boundary bowing, until the driving force for such bowing is balanced by the driving force which tends to decrease the grain boundary area. These authors are not aware of literature data for creep of polycrystalline materials which unequivocally can be interpreted in terms of elastic creep due to elastic anisotropy. Nevertheless, transient creep may possibly be attributed to this phenomenon. It is conceivable that such grain-boundary migration can be observed by measurement of internal friction at low frequency. Possibly, the observations of Kê [38] for the effect of grain boundaries on internal friction in part could be due to grain-boundary migration due to the elastic anosotropy.

The total creep strain which may result from grain-boundary migration in elastically anisotropic materials subjected to a given stress is expected to be of the order of elastic strain non-uniformities which result from the elastic anisotropy.

Elastic creep in composites by a redistribution of the individual phases requires simultaneous creep deformation of all phases. This will require mechanisms of creep in all phases which exhibit similar kinetics. This may occur in mixtures of polymers and metals with near identical melting points, crystal structures and other properties relevant to creep processes. For this reason, elastic creep by phase redistribution is unlikely to occur in the majority of composites such as fibre reinforced polymers or other materials in which the creep behaviour of the individual components exhibit marked differences. For such composites, deformation in one component may promote interface cracking such as observed by Birch and Wilshire [39] for Si_3N_4–SiC composites. In tension, this may possibly lead to elastic creep by crack growth. Because of the anticipated non-uniform stress distribution in such composites, an analysis of elastic creep by crack growth will probably have to rely on numerical (finite element) methods.

The total maximum elastic creep strain due to the redistribution of phases within a composite will be equal to the difference in elastic strain at initial loading and the elastic strain on complete elongation of the dispersed phases. For composites with large differences in the elastic properties of the components, the total elastic creep strain can be a large multiple of the initial elastic strain.

In a possibly subjective assessment of the relative importance of the four mechanisms of elastic creep considered in this study, elastic creep by crack and cavity growth and formation are more likely to be observed than elastic creep by grain-boundary migration or the redistribution of phases within composite materials.

A final remark is in order with regard to the measurement of creep deformation in which elastic creep by any mechanism is expected to be significant. For purposes of data analysis it is imperative that the relative contribution of the elastic creep to the total creep deformation is ascertained. As a minimum, this requires the measurement of the elastic recovery on removal of the load at the end of the creep experiment to be compared with the initial elastic strain. However, since the relative contribution of the elastic creep strain to the total creep strain is not expected to be linear with time or other creep displacement during the creep experiment, whenever, practical, any changes in elastic properties should be monitored continuously during the creep deformation. If the temperature level of the creep experiment permits, this could be performed most conveniently by ultrasonic or other acoustic techniques.

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